THE EFFECT OF A SHOCK WAVE ON A BURNING SOLID PROPELLANT*

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Abstract—A burning propellant at the end of a shock tube in the quiescent-gas zone sees a shock wave primarily as a pressure step. The burning rate is assumed to be surface-temperature-dependent. The time-dependent temperature distribution in the propellant is determined numerically. Burning-rate–time plots are presented for the variation of the important parameters. The effect of initial burning rate is discussed and it is suggested that this effect may bound the allowable frequency regions in solid-propellant combustion instability.

INTRODUCTION

Combustion instability in solid-propellant rocket motors has been a problem since World War II. All but the latest experimental and theoretical work has been recently reviewed(1). Unfortunately, past works have not indicated the direction to a solution of the problem, although the need for a solution has been increasing.

Recently there has been a fresh attack on the combustion-instability problem. R. W. Hart and F. T. McClure have examined "The Acoustic Interaction with a Burning Propellant Surface"(2) and have considered other problems of solid-propellant combustion instability(3,4). New and useful experimental data(5–8) have been presented, further defining the problem.

The experimental evidence is still not in total accord. The work of E. W. Price(5) on double-base propellants shows a negligible influence of pressure on combustion instability. Work at the Jet Propulsion Laboratory(7) on a polysulphide–ammonium perchlorate propellant have shown a pressure dependence. A sharp cut off was observed with stable combustion above and unstable combustion below some pressure; this critical pressure was frequency-dependent, increasing with decreasing

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frequency. The experimental data were analysed(8) in terms of a time-lag concept to obtain time-lag as a function of pressure, an analysis similar to that of Green(9).

The analysis of Hart and McClure indicated that it was highly probable that acoustic amplification would occur over a wide range of frequencies. In general, behaviour such as that obtained at the Jet Propulsion Laboratory was not predicted. The applicability of this theory to the JPL experiments cannot be so easily dismissed, because there are many other facets of the problem, all of which have been discussed before(2–4), and not the least of which is the acoustic impedance of the grain cavity.

It would be desirable to separate the propellant from its motor environment and perform some type of laboratory experiment. The ideal way to test the Hart and McClure thesis would be to measure the amplification of an acoustic wave impressed on a burning solid propellant. Neglecting other considerations, the expected amplification would be only 1–3 per cent; thus, an experimental determination would be difficult. It appears that it would also be difficult to determine whether a time-lag concept prevails through such an experiment. Alternatively, one might consider the effect of a sudden change in the pressure impressed on a burning solid propellant. This can easily be done in a shock tube. This paper considers the effect of a shock wave on a burning solid propellant.

The propellant is consider to be at the end of the shock tube in the reflected shock zone. The propellant and gaseous combustion zone see the shock wave only as a sudden change in the impressed pressure and the instantaneous solid-propellant burning rate. The burning rate is assumed to be an Arhennius function of the surface temperature. The instantaneous burning rate during the transient period following the shock wave can then be computed numerically. The calculated results of the effect of a number of parameters are presented. The effect of initial burning rate is shown; it is suggested that this effect may bound the allowable frequency regions in solid-propellant combustion instability.

THEORY

The equations to be considered are the differential equation of the temperature distribution in the solid propellant, for constant thermal diffusivity, where r is the propellant burning rate:

$$\alpha_s \frac{\partial^2 T}{\partial x^2} + r\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \theta} \tag{1}$$

the initial condition in the solid propellant:

$$T - T_i = (T_{s1} - T_i) \exp\,(-rx/\alpha_s) \tag{2}$$

and the boundary conditions, where H is a surface-heat-release term:

$$h(T_f - T_s) = -k_s\left(\frac{\partial T}{\partial x}\right)_0 + r\varrho_s H \tag{3}$$

and

$$T(\infty, \Theta) = T_i \tag{4}$$

Equation (2) is the steady-state solution of Eq. (1). At steady state,

$$h(T_f - T_s) = r\varrho_s c_s(T_s - T_i) + r\varrho_s H \tag{5}$$

and it can be noted that independent of the value of H, Eq. (2) holds, although the value of T_s will vary. Although the heat transferred from the combustion zone to the propellant is being represented by a heat-transfer coefficient and a temperature difference, this is just a method of representing the gradient at the surface, that is,

$$h(T_f - T_s) = -k_g\left(\frac{\partial T_g}{\partial x}\right)_0 \tag{6}$$

As will be brought out later, the use of Eq. (3) appears to be justified. The solution to Eq. (1) will be very depend upon what is happening in the gaseous combustion zone, and it is to this zone that most of the attention will be directed.

Consider the propellant to lie at the end of the shock tube in the reflected shock zone. Preliminary experiments have shown that when the propellant is in a flow zone the gas velocities behind the shock front may be sufficient to extinguish the propellant. The forward and reflected shock waves will have a velocity $\sim 10^5$ cm/sec and will pass through the combustion zone in $\sim 10^{-7}$ sec. The combustion zone will have momentary adiabatic heating, but will reach a quasi-steady state in $\sim 10^{-5}$ sec. (This value will be derived in the next paragraphs.) The adiabatic heating of the bulk gas will, of course, last much longer. However, burning rates obtained in a Crawford bomb and those obtained from motor firings have never shown any significant trends or differences despite the rather large differences in the bulk-gas temperatures. In fact, only recently[10] motor-determined burning rates were found to be 4 per cent lower than strand burning rates despite a careful radiation balance that indicated the motor-determined rates should be 4 per cent higher than the strand rates.

A steady-state equation can be written for the gaseous combustion zone:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(k_g \frac{\mathrm{d}T}{\mathrm{d}x}\right) - u\varrho_g a_g \frac{\mathrm{d}T}{\mathrm{d}x} = 0 \tag{7}$$

neglecting the small velocity effect and any heat produced by the exothermic combustion reaction. Assuming that k_g/c_g is constant, and taking appropriate solid-surface boundary conditions of equal temperatures and heat flux, the temperature distribution in the gas zone is

$$T = (c_s/c_g)(T_s - T_i)[\exp(r\varrho_s c_g x/k_g) - 1] + T_s \tag{8}$$

Using the values $c_s = 0.4$ cal/g°K, $c_g = 0.3$ cal/g°K, $\varrho_s = 1.64$ g/cm^3, $k_g = 2 \times 10^{-5}$ cal/cm sec °K, $T_i = 300$, $T_s = 700$, $T_f = 2160$°K and $r =$ 0.6 cm/sec, the thickness of the gaseous combustion zone δ is 9.5μ. If one assume a linear temperature gradient, $\delta = 18{\cdot}5\mu$, while the assumption of constant thermal diffusivity, and $\mu = r\varrho_s RT/PM$ gives $\delta = 7.5\mu$. The neglected of the exothermic combustion reaction should more than compensate for any errors in estimating the parameter values and $\delta \sim 10\mu$ should be reasonably accurate. Since, in general, propellant properties do not differ greatly, this value should be approximately correct for most conditions and propellants. This combustion-zone thickness should not be confused with apparent flame heights, which are quite misleading. And at the Jet Propulsion Laboratory, burning solid propellants have exhibited luminous flames with heights of the order $1000\mu^{(10)}$. These heights are probably a result of the distance required before the combustion products cool to a non-luminous condition. The oxidizer particle size and probably the surface roughness are also of the same order as δ, which does not ease the task of studying steady-state combustion. It will be assumed in this paper that the propellant surface is flat.

Assuming the gases to be ideal, the time for a particle to travel through the gaseous combustion zone is then

$$\tau = \int_0^\delta dy/u = \int_0^\delta (PM/r\varrho_s RT)\,dy \tag{9}$$

Substituting Eq. (8) in Eq. (9) and integrating yields

$$\tau = \frac{k_g PM}{r^2\varrho_s^2 R[c_s T_i - (c_s - c_g)T_s]} \ln\left\{\frac{T_s}{T_f}\left[1 + \frac{c_g(T_f - T_s)}{c_s(T_s - T_i)}\right] + 1\right\} \tag{10}$$

Using the values previously stated and $P = 42$ kg/cm^2 and $r = 0{\cdot}6$ cm/sec, the transit time is 5×10^{-6} sec. This selection of the values of P and r will be clarified later. The value of k_g is open to question; however, it does not appear to have been underestimated by more than a factor of two, and τ should in no case be greater than 10^{-5} sec. The characteristic time associated with the conduction of heat$^{(2)}$ is also $\sim 10^{-5}$ sec under these conditions. In brief, following the instantaneous pressure step, any external influences or those caused by the shock wave can be neglected after 10^{-5} sec. It can be noted, since one usually equates the equilibrium burning rate and pressure by $r = aP^n$, that $\tau \sim P^{1-2n}$, and that for the usual range for n of 0.2–0.6, normal variations in pressure will only have a small effect on the transit time. At constant pressure, $\tau \sim r^{-2}$; however the calculated results will show that the transient-process time is always much longer than τ.

This discussion of the combustion zone will be interrupted for a moment while an analytic solution to Eq. (1) is considered. The assumption that all the parameters in Eqs. (1–4) are constant allows one to obtain a nontrivial solution to Eq. (1). As the solution is rather long and not of great interest, it will not be reproduced here; however, two of the results obtained will be considered.

The ratio of the surface temperature to the initial surface temperature as a function of time is presented in Fig. 1 for two cases. The first involves the instantaneous change from h_1 and r_1 to h_2 and r_2. The changes were selected to keep T_s constant although h_2 and r_2 could be selected to vary T_{s2} at will. The second case only changes h and, therefore, changes T_s. In later numerical calculations, the burning rate is considered to be an Arhennius function of the surface temperature. In the analytical results it was thought that the two cases should be similar to high an low activation energies respectively in the varying-burning-rate calculation.

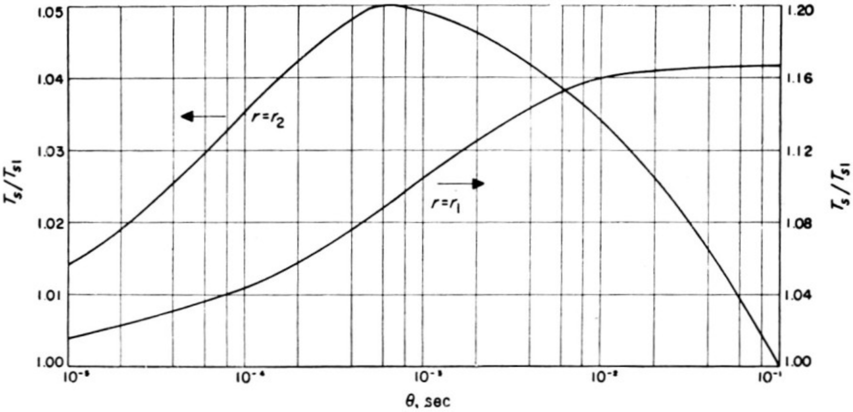

FIG. 1. Results of analytical solution.

It can also be seen in Fig. 1 that a reasonable amount of the change in T_s takes place after 10^{-3} sec. In the actual case, this change in T would also be expected to result in burning-rate changes. In the second case illustrated (low activation energy), the burning rate would actually increase; therefore, it was expected that a greater portion of the change would take place after 10^{-3} sec than indicated in the figure. In the first case (high activation energy), the rather small change in T_s would result in large burning-rate changes, presumably also increasing the time to reach equilibrium. As it appeared that burning-rate changes occurring after 10^{-3} sec might be determined experimentally, a small experimental programme was started and the numerical calculations discussed in the following paragraphs were made. As will be seen, the numerical calculations indicate that for the case of interest the transient process is essentially complete by 10^{-3} sec, amplifying tremendously the difficulties of any experimental programme. The solutions of Eq. (1) in which r and perhaps h are allowed to vary must be performed numerically. It was assumed that the burning rate was $r = A \exp(-E/RT_s)$. This assumes the burning rate is controlled by a surface reaction or desorption, as has previously been assumed(12). More recently(2), it has been assumed that the burning rate is volume controlled,

$$r = \int A \exp(-E/RT)\,\mathrm{d}x \tag{11}$$

Calculations to determine the effect of this latter model are planned.

The propellant of initial interest is one on which a considerable amount of rocket-motor combustion instability research has been done(7,8). It is a polysulphide–ammonium perchlorate propellant of medium impulse, with an average c^* of 1290 m/sec at 71°C. The adiabatic combustion products have the properties $\gamma = 1.25$, $M = 25$ and $T_f = 2160°\text{K}$. The non-erosive-burning data are given in Fig. 2. One can determine $h = f(P,r)$ from this data. The burning rate is a function of the initial propellant temperature at any one pressure. Figure 3 illustrates this at 42 kg/cm^2.

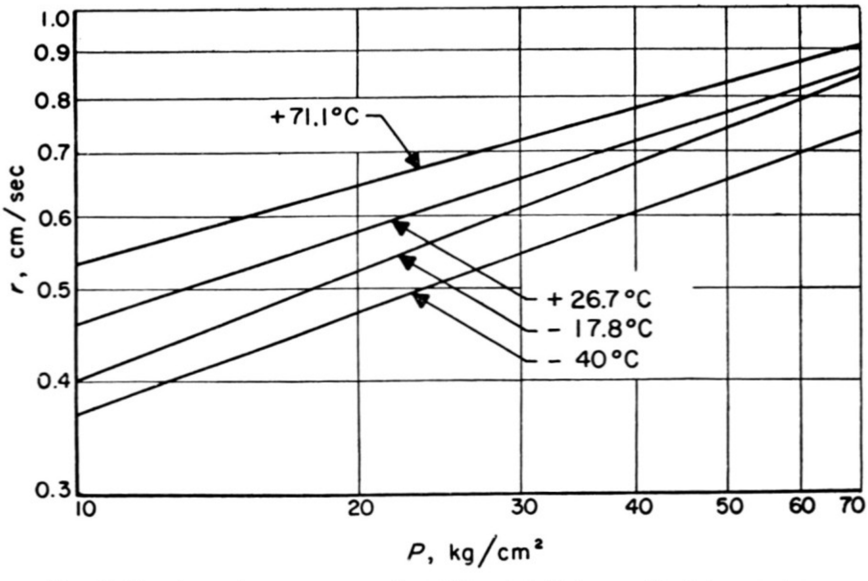

FIG. 2. Burning rate vs. pressure for different initial propellant temperatures.

It was assumed that $T_s = 700°\text{K}$ at $P = 21$ kg/cm², $T_i = 26.7°\text{C}$ and $r = 0.585$ cm/sec. The value of T_s was based on measurements presently being made at Jet Propulsion Laboratory. Figure 4 then present h vs. r at $P = 42$ kg/cm², based on different assumed values of the activation energy E.

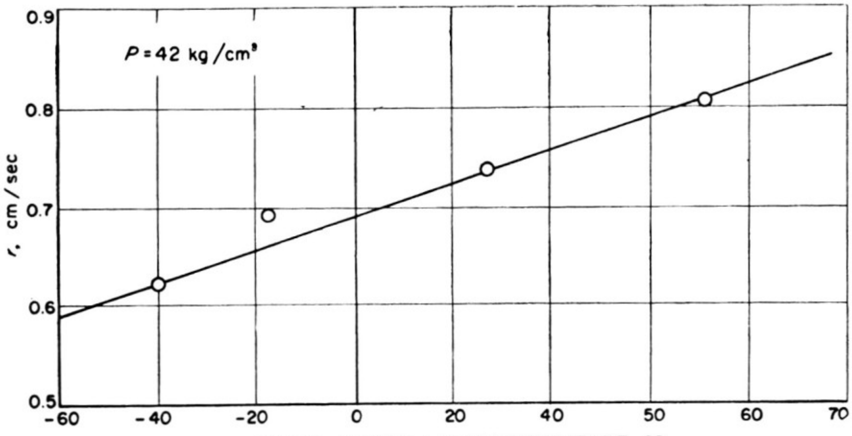

FIG. 3. Burning rate vs. propellant temperature.

The relationships shown in Fig. 4 are based on steady-state conditions. The gaseous combustion should be independent of the temperature gradient within the propellant and a function only of P and r. It has been

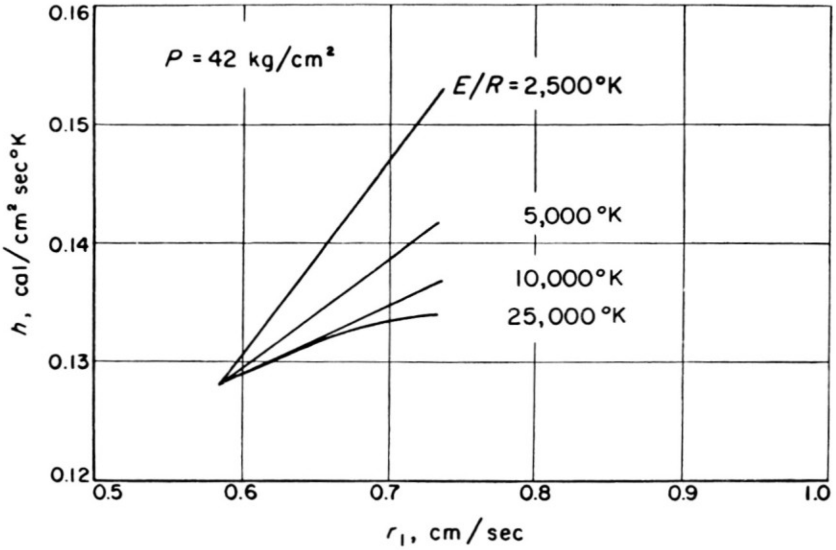

FIG. 4. Heat-transfer coefficient vs. burning rate.

shown in previous paragraphs that for the case of initial interest the gas zone should reach a quasi-steady-state condition in about 10^{-5} sec. The relationships in Fig. 4 should hold for the transient process following the impact of the shock wave, provided the overall times are larger than 10^{-5} sec. The analytical solution and the calculations to be presented show that this assumption appears reasonable.

RESULTS OF CALCULATIONS

The results of Fig. 4 indicated that the variation in h during the transient process should be small. It was expected that during the transient process r would not greatly exceed r_2. Therefore, the calculations performed to date were done with constant h. The value of h at P_2 and r_2, h_2, was used. The use of constant h appears to be sufficiently accurate for the present.

The cases of high E show a large change in burning rate in times near τ and do not fit the proposed model very well. An accurate solution for these cases would require an analysis of the initial effect of the shock wave. However, the initial adiabatic heating of the gas zone should compensate somewhat for the lower heat flux during the time required to reach the quasi-steady-state condition and the calculations should still be reasonably correct. Errors should consist mainly of a shift of about τ sec in the time scale.

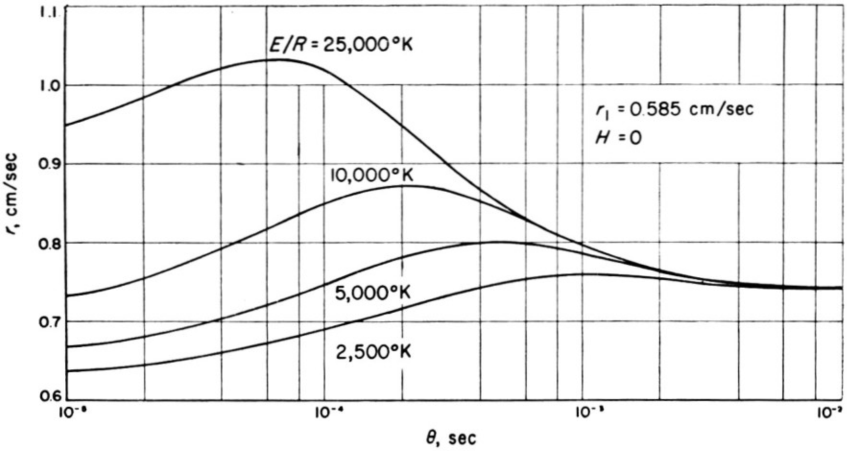

FIG. 5. Time dependence of burning rate following shock.

Equation (1) was solved on an IBM 704. Time increments as small as 10^{-6} sec were used. There were minor difficulties because of the exponential temperature profile (Eq. 2). A transformed equation in which the distance variable provided a linear temperature or temperature gradient would have been easier to solve numerically.

Representative results of the variation of burning rate and surface temperature with time are presented in Figs. 5–9. The variables investigated were E, r_1, H and h_2. The initial surface temperature T_{si} was 700° K for all cases. Normal variations in T_i, T_{si}, T_f, ϱ_s, c_s and k_s should have a negligible effect.

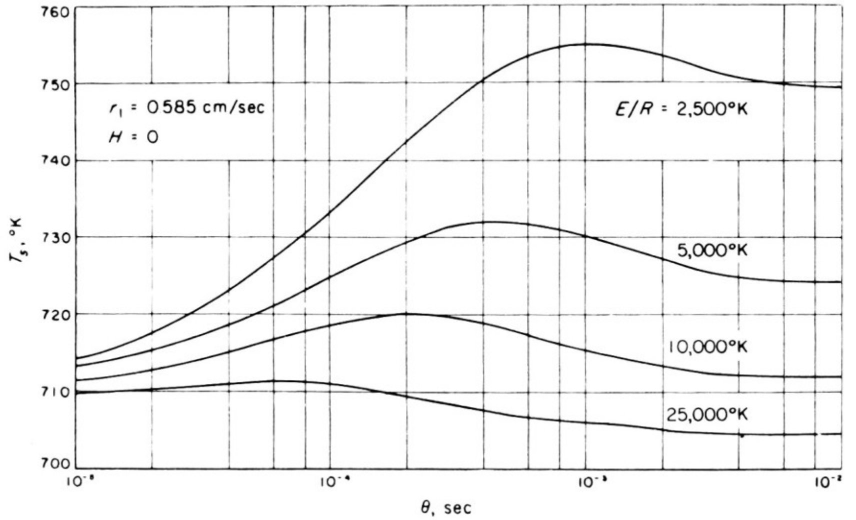

FIG. 6. Time dependence of surface temperature following shock.

The first calculations corresponded to the experimental condition on the propellant discussed earlier. These were $P_1 = 21$ kg/cm^2 and $P_2 = 42$ kg/cm^2 corresponding to $r_1 = 0.585$ cm/sec, $r_2 = 0.736$ cm/sec, and $h_2 =$

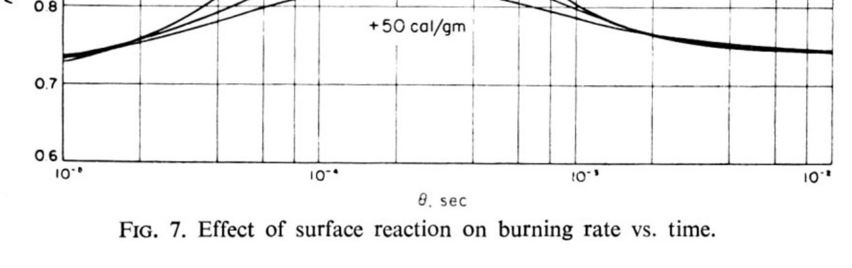

FIG. 7. Effect of surface reaction on burning rate vs. time.

0,1047 cal/g cm^2 °K. This pressure step is higher than the usual oscillating pressure in unstable solid-propellant rocket motors. This was a compromise selected to yield a reasonable change in burning rate without an excessive pressure step. Higher-pressure shocks would also have required a longer tube to maintain the same time duration.

The time-dependency of burning rate under these conditions is presented as a function of activation energy in Fig. 5. The results are in qualitative agreement with the analytical results of Fig. 1. The case of high E (Fig. 5) shows a larger change in r and reaches its maximum sooner that the low-E case. The transient process is essentially completed by 10^{-3} sec. The effect of activation energy on T_s is inverted, as expected, and is shown in Fig. 6.

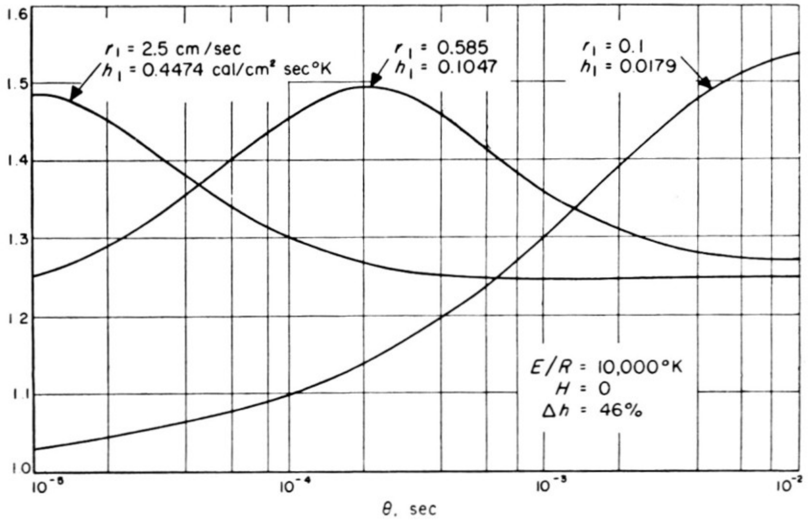

FIG. 8. Effect of initial burning rate on burning rate vs. time.

The effect of a surface reaction is shown in Fig. 7 at $E \sim 20$ kcal/mol. This is a reasonable estimate for the decomposition of ammonium perchlorate$^{(13)}$. An exothermic ($-H$) surface reaction has a much greater effect than an endothermic ($+H$) reaction. The effect of a variation in H increases with increasing E and was almost negligible at the lowest E calculated, $E/R = 2500°$ K.

Results similar to those shown were found for $r_1 = 0.1$ and 2.5 cm/sec, although the time scale is different and is approximately $\sim r^{-2}$. The ratio v/v_1 for three different values of r_1 is shown in Fig. 8. It appears that the results might superimpose if plotted on a t/r_1^2 time scale.

The variation in r was almost directly proportional to the variation in h. Changes in h had a negligible effect on the time scale. The fractional increase in r as a function of the fractional increase in h was only slightly, if at all, depend on r_1. The effect of the variation of h on r_m/r_1, where r_m is the maximum burning rate, is shown in Fig. 9. The curve represents the averaged results for the three different burning-rate cases.

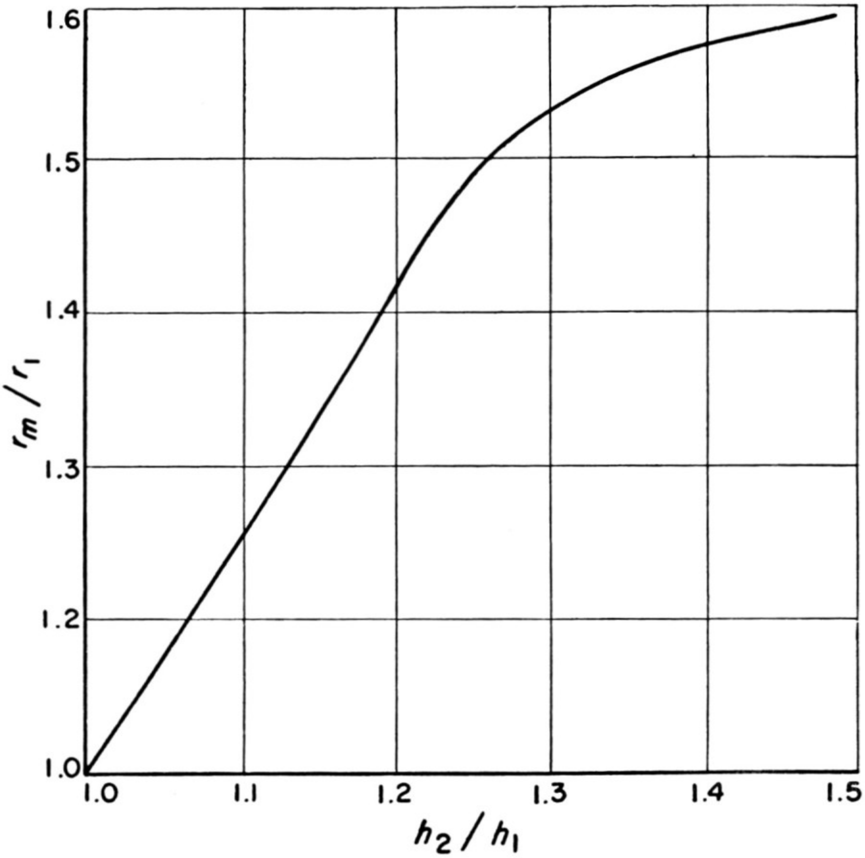

FIG. 9. Maximum burning rate vs. increase in heat-transfer coefficient.

The results of the calculations indicate that for the case of present interest ($r_1 = 0.585$ cm/sec) the transition is essentially completed by 10^{-3} sec; it will thus be difficult to obtain experimental data. It appears that a lower burning rate, obtained by either lower pressure or a change in propellant, and a stronger shock would provide a more suitable experiment. The required shock durations of 10–100 msec are somewhat difficult to obtain, but because of the relatively long time, the pressure step might be obtained in some simpler manner.

It will also be difficult to determine whether a time-lag concept prevails. A previous analysis(8) calculated time lags from 10^{-5} to 10^{-4} sec for pressure from 14 to 70 kg/cm^2. These were attributed to the gas-phase combustion at that time. At burnt-gas velocities these lags are equivalent to a displacement of 10 to 20 combustion zone thicknesses. The time-lag could have easily been in the solid phase, as suggested by Green(9).

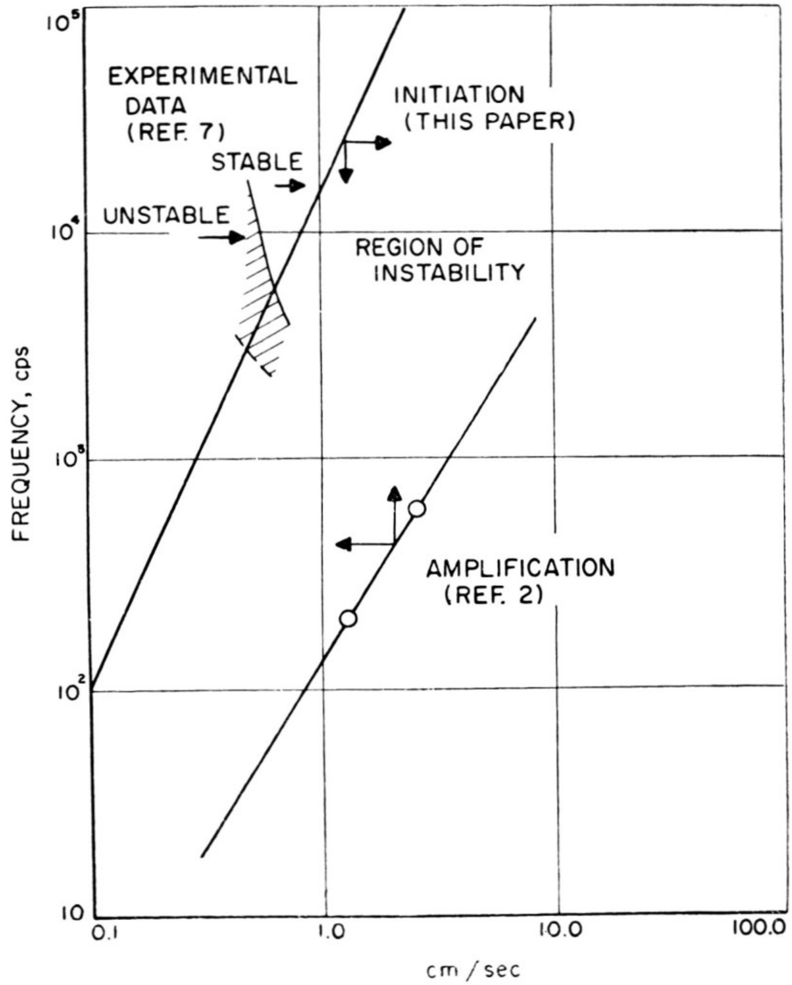

FIG. 10. Combustion-instability frequency dependence on burning rate.

In either case, because of the small value of the time-lag, it is difficult to see how the inclusion of a time-lag in the model proposed here would yield significantly different results.

Some rather broad conclusions may be drawn from the calculations presented. Green(14) suggested that the severity of instability should depend on the energy flux, burning rate times heat content, and this has been experimentally demonstrated by Price⁽⁵⁾. The results of these calculations indicate that fast-burning propellants have a quicker response to disturbances than slow-burning propellants and therefore would probably show more severe instability than the slower-burning propellants. This dependence of the amplitude of the response on the burning rate was also shown in the analysis previously made⁽⁸⁾.

It also appears that a disturbance of longer time would be required to initiate instability in a slow-burning propellant, than in a fast-burning one. If the times are arbitrarily related to frequency, then the initiation of instability in the slower-burning propellant, for example, would be more probably at frequencies less than 100 c/s. One then obtains a region of more probable initiation in the frequency-burning-rate plane as shown in Fig. 10. The frequency over which amplification takes place is also burning-rate dependent, and the two points calculated⁽²⁾ are shown and liberally extrapolated.

Thus, the instability zone has been bounded, although many variables have not been considered and the boundaries are much more vague than indicated. The results of some experimental data⁽⁷⁾ are also presented, which, unfortunately, do not conform to the theories.

Preliminary experiments utilizing a shock tube and high-speed photography were run in an attempt to measure the instantaneous burning rate immediately following the shock wave. Difficulties were encountered with tube movement, smoke and insufficient lighting. The camera-film resolution was only 35 lines/mm, and is the major problem. An improvement to 50 lines/mm may be possible. Calculations are being made with $r_1 = 0.1$ cm/sec to determine the pressure step that will give a burning rate above 0.5 cm/sec during the transient. If the transient occurs at the times indicated in Fig. 8 an experimental measurement appears feasible. Otherwise, it appears that experimental measurements will be difficult to obtain.

SUMMARY

A laboratory experiment was sought that would yield data helpful in solving the solid-propellant combustion-instability problem. Subjecting a burning propellant to a shock wave and measuring the instantaneous burning rates during the transient period appeared to be a reasonable experiment. The propellant is mounted at the end of the shock tube in the quiescent-gas zone. The propellant sees the shock primarily as a sudden step in the impressed pressure. The temperature distribution in the propellant is solved numerically, assuming the burning rate is surface-temperature dependent. Burning-rate-time plots are presented for the variation of a number of parameters. It is suggested that the effect of burning rate on the response of the propellant to an impressed pressure may bound the allowable frequencies of combustion instability.

It is concluded that it will be difficult to obtain experimental measurements.

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